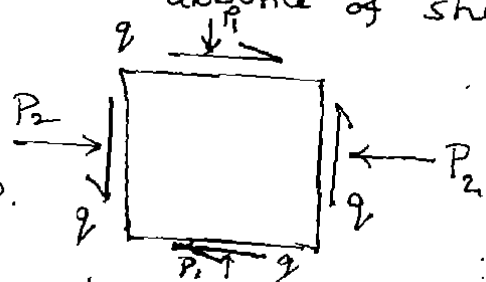


Differences between Open channel Flow and Pipe flow

Open channel Flow	Pipe Flow
1. Open to atmosphere (Pressure is zero)	1. Pressures other than zero (may be +ve or -ve)
2. Flow of open channel is gravitational i.e., from high level to low level.	2. Pipe flow occurs from high total head to low total head. It may be from low level to high level or high level to low level.
3. Water surface is the HGL, T.E.L of open channel is above HGL, with an ordinate of corresponding velocity head.	3. In case of pipes, HGL may be above the pipe line or below pipe line Eg: Syphon.
4. Velocity distribution is logarithmic	4. Velocity distribution may be parabolic (laminar) or almost rectangular (turbulent)
5. Shapes can be semicircular, trapezoidal, Rectangular	5. Usually circular c/s

⇒ Atmospheric pressure is uniform on the free surface.
According to pascal's Law, pressure is uniform in all directions at a point in absence of shear stress only.

∴ If a stress element is a
shown, for uniform pressure $p_1 = p_2 = p$.
 $\tau = 0$

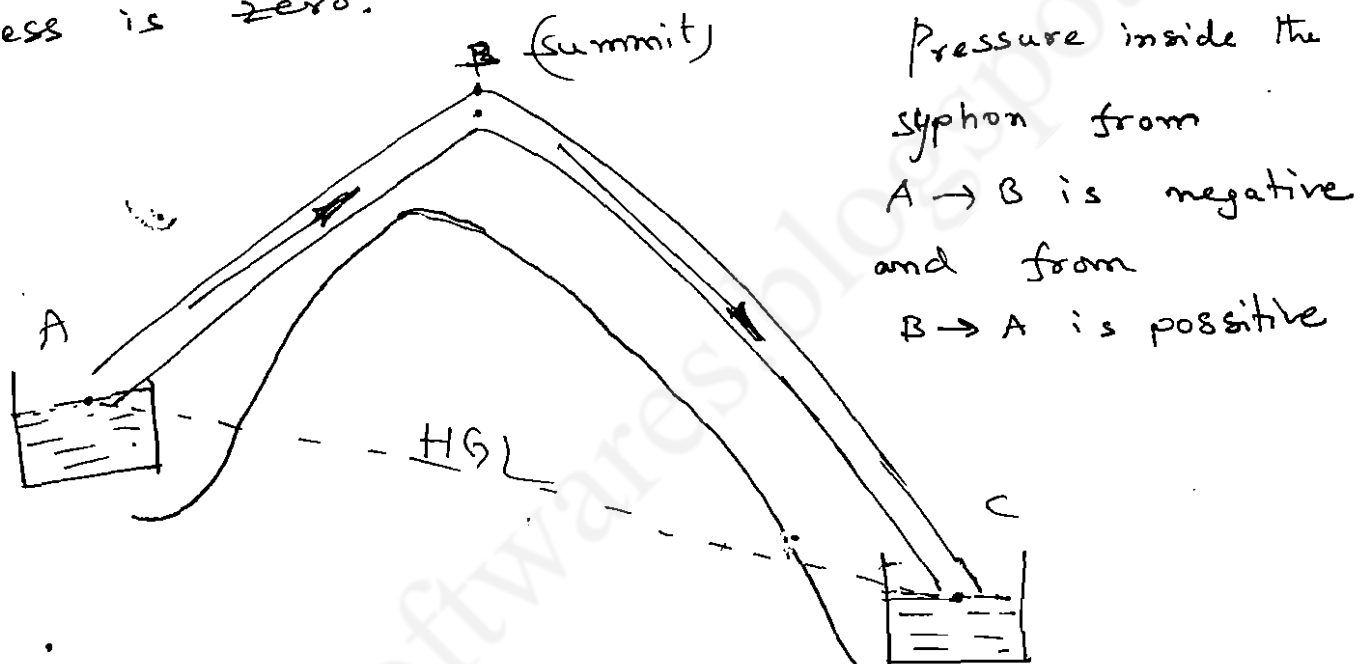


Radius of Mohr circle, $r = \sqrt{\left(\frac{P_1 - P_2}{2}\right)^2 + q^2}$

$$= 0$$

∴ The Mohr circle for uniform pressure at a point is a point on the normal stress axis.

A free surface is subjected to uniform atmospheric pressure. Uniform pressure is possible only if shear stress is zero.



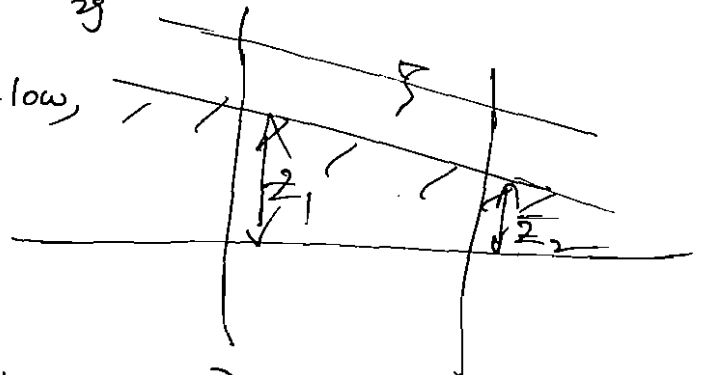
⇒ for a pipe flow, pressures other than zero

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h$$

for constant depth of flow,

$$V_1 = V_2$$

$$P_1 = P_2 = P_{atm}$$



∴ $Z_1 > Z_2$ (gravitational flow)

UNIT-I

Open channel Flow-I

Learning Objectives :

At the end of this topic you will be able to :

- ⇒ Types of flow
- ⇒ Types of channels
- ⇒ Velocity distribution in a channel section
- ⇒ Energy and correction factors
- ⇒ Discharge through open channel by chezy's formula
- ⇒ chezy's constant
- ⇒ Most economical cross-section for open channels
- ⇒ specific energy curve
- ⇒ Types of critical flow.

Outcomes

By the end of this topic you will be able to :

- ⇒ Describe the various types of flows through open channel.
- ⇒ Explain the various types of channels and their functions.
- ⇒ Estimate the level of velocity distribution in a channel section.
- ⇒ Explain the major function of energy and correction factors
- ⇒ Find out the discharge through open channel by chezy's formula.
- ⇒ Differentiate the method of chezy's constant
- ⇒ Describe the most economical cross-section for open channels.

Subject Overview

Why are we studying this subject

To understand

- What is an open channel?
- How does the channel plays the major role in velocity distribution?
- How to find out the most economical channel sections?
- Implementation of a correction factors through the over all energy process?
- What is the impact of froude's number in the entire channel flow?
- Differentiate the various methods of the dimensional analysis?
- How does the force exerted by fluid jet on stationary and moving normal and an inclined flat plate?
- The performance characteristics of hydraulic turbines!

Relation of the concepts with other Engineering Branches

The concept of

- The term steady and uniform are used frequently in Engineering
- Consideration of laminar & turbulent flow in Numerical Methods.
- One, two and three-dimensional flows concepts are used in the Engineering hydraulics.
- Force exerted by fluid jet concepts have been used in Thermal and hydraulic machines.

→ Typical Hydro power plant procedures are used in Power plant engineering

→ The method of dimensional analysis is used in every fields, especially in such fields as Fluid dynamics and thermodynamics.

Relationship between the concepts in the module.

→ The topic of Open channel flow will gives a brief view about the difference between the natural and artificial channel flows and also able to analyze the variation of flows which applied in the flow distribution channels.

Types of flow

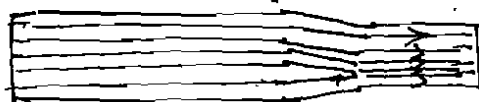
The Flow in channels can be classified into following Types depending upon the change in the depth of flow with respect to space and time.

Steady flow :- A steady flow is one in which all conditions at any point in a stream remains constant with respect to time.

$$\left(\frac{\partial V}{\partial t} \right)_{x_0, y_0, z_0} = 0 \quad \left(\frac{\partial p}{\partial t} \right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial \rho}{\partial t} \right)_{x_0, y_0, z_0} = 0$$

Where

(x_0, y_0, z_0) is a fixed point in fluid field



Unsteady flow : A flow in which quantity of liquid flowing is not constant, then it said to be as unsteady flow.

$$\left(\frac{\partial v}{\partial t} \right)_{x_0, y_0, z_0} \neq 0, \quad \left(\frac{\partial p}{\partial t} \right)_{x_0, y_0, z_0} \neq 0$$

Uniform flow : Uniform flow is one in which the depth, discharge, mean velocity etc. do not change along the channel at any given instant.

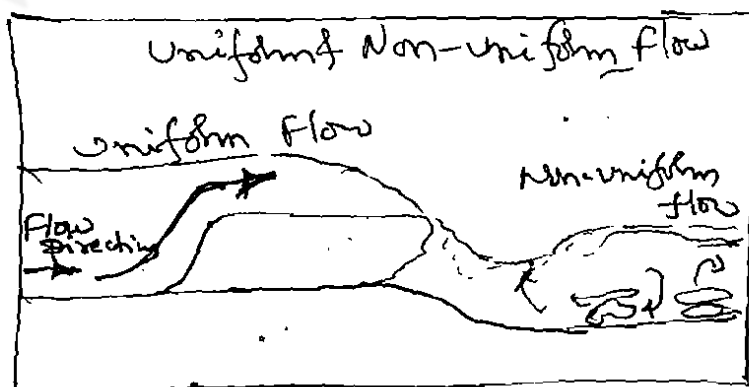
$$\left(\frac{\partial v}{\partial s} \right)_{t=\text{constant}} = 0$$

where ∂v = change of velocity

∂s = Length of flow in the direction s .

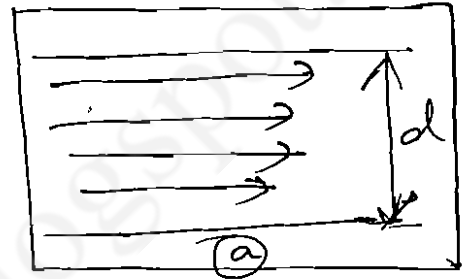
Non-Uniform flow : Non-uniform flow is one in which the depth, discharge, mean velocity etc. can change along the channel at any given instant.

$$\left(\frac{\partial v}{\partial s} \right)_{t=\text{constant}} \neq 0$$



Laminar Flows:

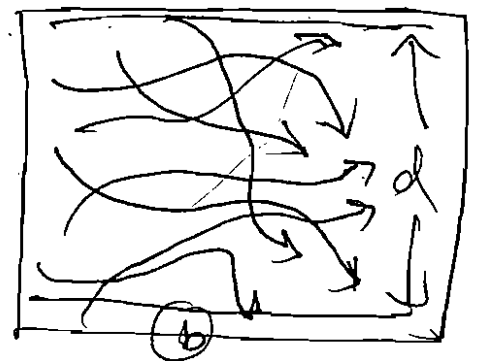
- Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the streamlines are straight and parallel.
- Thus the particles move in laminae & layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or, viscous flow.



Turbulent Flow:

- Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way.
- Due to the movement of fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss.

- For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number, where $D = \text{Diameter of Pipe}$



$V = \text{Mean velocity of flow in pipe}$ &
 $\nu = \text{Kinematic viscosity of fluid.}$

→ If the Reynold number is less than 2000, the flow is called laminar.

→ If the Reynold number is more than 4000, it is called turbulent flow.

→ If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

Compressible Flow :

→ Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid.

$$\rho \neq \text{constant}$$

Incompressible Flow :

→ Incompressible flow is that type of flow in which the density is constant for the fluid flow, liquids are generally incompressible while gases are compressible.

$$\rho = \text{constant}$$

Rotational Flows :

→ Rotational Flow is that type of flow in which the fluid particles while flowing along streamlines, also rotate about their own axis.

Irrrotational Flows:

→ If the fluid particles while flowing along streamlines, do not rotate about their own axis that type of flow is called irrotational flow.

One, Two and Three - Dimensional Flows.

One dimensional Flows:

→ All the flow parameters may be expressed as functions of time and one space coordinate only.

→ The single space coordinate is usually the distance measured along the centre-line (not necessarily straight) in which the fluid is flowing.

$$u = f(x), \quad v = 0 \quad \& \quad w = 0.$$

Two dimensional Flows:

→ All the flow parameters are functions of time and two space coordinates (say x and y).

→ No variation in z direction.

→ The same streamline patterns are found in all planes perpendicular to z direction at any instant.

$$u = f_1(x, y), \quad v = f_2(x, y) \quad \& \quad w = 0.$$

Three - dimensional Flows:

→ The hydrodynamic parameters are functions of three space coordinates and time.

$$u = f_1(x, y, z), \quad v = f_2(x, y, z) \quad \text{and} \quad w = f_3(x, y, z).$$

Classification of channels :- classification Type 1.

1. Natural channels :- Channels exist naturally on the earth.
ex: Rivers, tidal estuaries, ^{streams}, etc. Irregular shape.
2. Artificial channels: channels developed by men. They are regular geometric shapes.
ex: Irrigation canals, laboratory flumes, spillway chute drops, culverts, roadside gutters etc.

Classification Type 2

1. Prismatic channels :- A channel with unvarying C/S and the constant bottom slope is called prismatic channel. All the artificial channels are usually prismatic. The rec., trapezoid, parabola & circle are the most common used shapes of prismatic channels.
2. Non-prismatic channels:- A channel with varying cross-section and the constant bottom slope is called non-prismatic channel. The natural channels are usually prismatic.

Classification Type 3 :-

1. Rigid Boundary channels: A channel with immovable bed & sides is known as rigid boundary channels.
ex: Lined canals, sewers and non-erodible ^{lined} canals.
2. Mobile Boundary channels: If a channel boundary is composed of loose sedimentary particles moving under the action of flowing water. ex: Alluvial channel is transporting same type of material.

Classification Type 4

1. Small slope channels:

An open channel having a bottom slope less than 1 in 10 is called a channel of small slope (Chow, 1959). The slopes of ordinary channels, natural & artificial are less than 1 in 10.

2. Large slope channels:- An open channel having a bottom slope greater than 1 in 10 is called a channel of large slope (Chow, 1959). Some artificial channels like drops & chutes have far more than 1 in 10.

Velocity distribution in a channel section:-

→ The flow velocity in a channel section varies from one point to another. This is due to shear stress at the bottom and at the sides of the channel and due to the presence of free surface.



Triangular channel



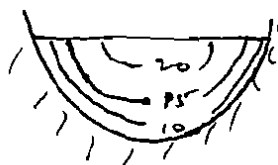
Trapezoidal channel



Natural irregular channel



Pipe



Shallow ditch



Narrow rectangular section

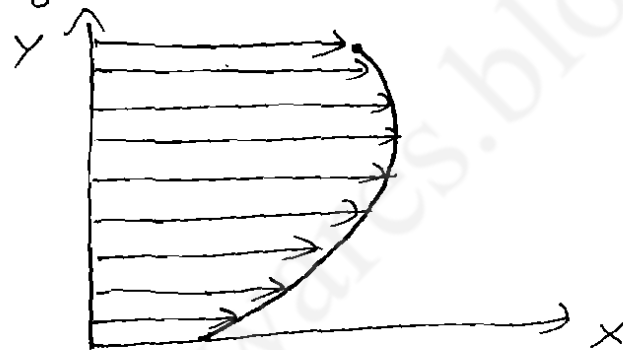
② Velocity distribution in different channel section

→ Figure (a) shows typical velocity distributions in different channel cross sections.

→ The flow velocity may have components in all three Cartesian coordinate directions.

→ However, the components of velocity in the vertical and transverse directions are usually small and may be neglected. Therefore, only the flow velocity in the direction of flow needs to be considered.

→ This velocity component varies with depth from the free surface. A typical variation of velocity with depth is shown in figure (b).



(b), Typical velocity variation with depth

Energy and ^{Momentum} Correction factors:

→ When energy and momentum principles are used in open channel flows, it has been customary to compute the velocity head and momentum flux of the flow from the average velocity.

→ In reality, the velocity distributions are not uniform over the cross section and hence the velocity head and the momentum flux of open channel flow are generally greater than the values computed ^{by} using the average velocity.

- These values may be corrected by using so-called energy and momentum correction coefficients, which are always slightly greater than the limiting value of unity.
- The flux of the kinetic energy flowing past a section can also be expressed in terms of V . But in this case, a correction factor α will be needed as the kinetic energy per unit weight $\frac{V^2}{2g}$ will not be the same as $\frac{V^2}{2g}$ averaged over the cross-section area.
- An expression for α can be obtained as follows:
- For an elemental area dA , the flux of kinetic energy through it is equal to

$$= \left(\frac{\text{mass}}{\text{time}} \right) \left(\frac{\text{KE}}{\text{mass}} \right) = \left(\rho V \cdot dA \right) \frac{V^2}{2}$$

For the total area, the kinetic energy flux

$$= \int_A \frac{\rho}{2} V^3 dA = \alpha \cdot \frac{\rho}{2} V^3 \cdot A$$

$$\text{For which } \alpha = \frac{\int V^3 dA}{V^3 A}$$

$$\text{or for discrete values of } V, \alpha = \frac{\sum V^3 \Delta A}{V^3 A}$$

α is known as the kinetic energy correction factor and is equal to or greater than unity.

The kinetic energy per unit weight of fluid can then be written as $\alpha \frac{V^2}{2g}$.

Momentum :-

→ Similarly, the flux of momentum at a section is also expressed in terms of V and a correction factor β . Considering an elemental area dA , the flux of momentum, in the longitudinal direction through this elemental area

$$= \left(\frac{\text{mass}}{\text{time}} \times \text{velocity} \right) = \rho v dA (v)$$

For the total area, the momentum flux

$$= \int \rho v^2 dA = \beta \rho v^2 A$$

which gives $\beta = \frac{\int_A v^2 dA}{V^2 A} = \frac{\sum V^2 \Delta A}{V^2 A}$

β is known as the momentum correction factor and is equal to or greater than unity.

Discharge through Open channel by Chezy's formula,

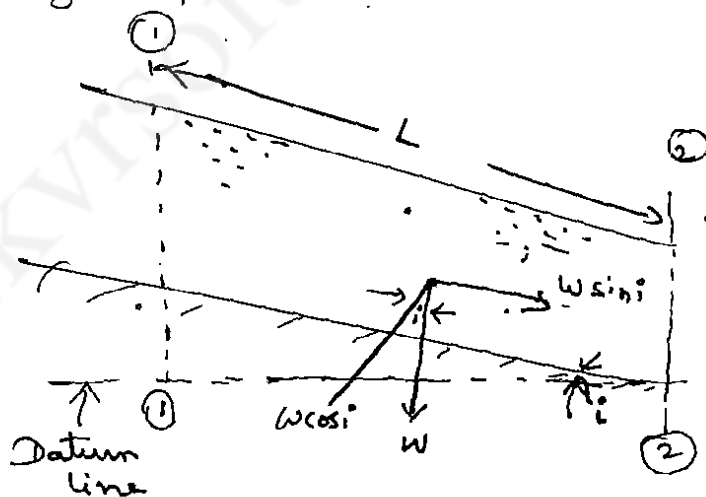


fig: Uniform flow in Open channel

→ Consider uniform flow of water in a channel as shown in figure. As the flow is uniform, it means the velocity, depth of flow and area of flow will be

be constant for a given length of the channel.

consider sections 1-1 and 2-2.

Let L = length of channel

A = Area of flow of water

i = slope of the bed.

V = Mean Velocity of flow of water.

P = Wetted perimeter of the cross-section.

f = Frictional resistance per unit velocity per unit area.

The weight of water between sections 1-1 and 2-2.

W = specific weight of water $\times$ Volume of water

$$= w \times A \times L$$

Component of W along direction of flow = $W \times \sin i$

Frictional resistance against motion of water
 $= w \times A \times L \times \sin i$
 $= f \times \text{Surface area} \times (\text{Velocity})^n$

The value of n is found experimentally equal to 2
and surface area = $P \times L$.

frictional resistance against motion = $f \times P \times L \times V^2$.

→ The forces acting on the water between sections 1-1 and 2-2 are:

- i) Component of weight of water along the direction of flow
- ii) frictional resistance against flow of water.
- iii) pressure force at section 1-1
- iv) pressure force at section 2-2.

→ As the depths of water at the sections 1-1 and 2-2 are the same pressure forces on these two sections are same and acting in the opposite direction. Hence they cancel each other.

→ In case of uniform flow, the velocity of flow is constant for the given length of the channel ~~each~~ ~~other~~. Hence there is no acceleration acting on the water.

→ Hence the resultant force acting in the direction of flow must be zero.

∴ Resolving all forces in the direction of flow, we get

$$wAL \sin i = f \times P \times L \times V^2 = 0$$

$$V^2 = \frac{wAL \sin i}{f \times P \times L} = \frac{w}{f} \times \frac{A}{P} \times \sin i$$

$$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P}} \times \sqrt{\sin i}$$

But $\frac{A}{P} = m$
= hydraulic mean depth (or) hydraulic radius

$$\sqrt{\frac{w}{f}} = C = \text{Chezy's constant}$$

Substituting the values of $\frac{A}{P}$ and $\sqrt{\frac{w}{f}}$ in eq, $V = C \sqrt{m i}$

for small values of i , $\sin i = \tan i = i$ ∴ $V = C \sqrt{m \sin i}$

$$\begin{aligned} \therefore \text{Discharge, } Q &= \text{Area} \times \text{Velocity} = A \times V \\ &= A \times C \sqrt{m i} \end{aligned}$$

1. Find the velocity of flow and rate of flow of water through a rectangular channel of 8m wide and 2m deep. when it is running full. The channel is having bed slope as 1 in 2500. Take Chezy's constant $C = 65$.

Given Data:

Width of rectangular channel, $b = 8\text{m}$

Depth of channel, $d = 2\text{m}$

Area, $A = 8 \times 2 = 16\text{m}^2$

Bed slope, $i = 1 \text{ in } 2500 = \frac{1}{2500}$

Chezy's constant, $C = 65$

Perimeter, $P = b + 2d = 8 + 2 \times 2 = 12\text{m}$

Hydraulic mean depth, $m = \frac{A}{P} = \frac{16}{12} = 1.33\text{m}$

To Find:

Velocity of flow and rate of flow of water.

Formula used:

Velocity of flow, $V = C \sqrt{mi}$

Rate of flow, $Q = V \times A$

Solution:

Velocity of flow is given by equation as

$$V = C \sqrt{mi} = 65 \sqrt{1.33 \times \left(\frac{1}{2500}\right)} = 1.49 \text{ m/s}$$

$$\text{Rate of flow, } Q = A \times V$$

$$= 16 \times 1.49$$

$$= 23.98 \text{ m}^3/\text{s}$$

∴ Result:

$$\text{Velocity of flow, } V = \underline{1.49} \text{ m/s.}$$

$$\text{Rate of flow, } Q = \underline{23.98} \text{ m}^3/\text{s}.$$

2. A Flow of water of 100 litres per second flows down in a rectangular flume of width 600mm and having adjustable bottom slope, if chezy's constant C is 56. Find the bottom slope necessary for uniform flow with a depth of flow of 300mm. Also find the conveyance K of the flume.

Given Data:

$$\text{Discharge, } Q = 100 \text{ litres/s} = \frac{100}{1000} = 0.10 \text{ m}^3/\text{s}$$

$$\text{Width of channel, } b = 600 \text{ mm} = 0.6 \text{ m.}$$

$$\text{Depth of flow, } d = 300 \text{ mm} = 0.3 \text{ m.}$$

$$\text{Area of flow, } A = b \times d = 0.6 \times 0.3 = 0.18 \text{ m}^2$$

$$\text{Chezy's Constant, } C = 56.$$

To find: Bottom slope
Conveyance k of the flume.

Formula used:

$$k = AC\sqrt{m} \quad , \quad Q = AC\sqrt{mi}$$

Solution:

Let the slope of bed = i

$$\text{Hydraulic mean depth, } m = \frac{A}{P} = \frac{0.18}{0.6 + 2 \times 0.3} = 0.15 \text{ m.}$$

Squaring both sides, we have

$$0.15 i = \left(\frac{0.10}{0.18 \times 56} \right)^2$$

$$i = \frac{0.00098418}{0.15} = 0.006512$$

$$= \frac{1}{\frac{1}{0.0006512}} = \frac{1}{1524}$$

$\therefore$ Slope of the bed is 1 in 1524.

Conveyance k of the channel

$$\therefore \text{Discharge, } Q = AC\sqrt{mi}$$

which can be written as $Q = k\sqrt{i}$.

Where $k = AC\sqrt{m}$ & k is called Conveyance of the channel section.

$$k = 0.18 \times 56 \times \sqrt{0.15} = 3.9039 \text{ m}^3/\text{s}$$

Result:

Conveyance of the flume $k = 3.9039 \text{ m}^3/\text{s}$.

Bottom Slope of channel, $i = 1$ in 1524.

Chezy's Constant:-

Chezy's formula after the name of a French Engineer. Antoine Chezy who developed this formula is 1975. In this equation C is known as Chezy's constant, which is not a dimensionless co-efficient. The dimension

$$\begin{aligned} \text{of } C \text{ is } &= \frac{V}{\sqrt{mi}} = \frac{L/T}{\sqrt{\frac{A}{P} i}} = \frac{L/T}{\sqrt{\frac{L^2}{L} i}} = \frac{L}{T\sqrt{L} i} = \frac{\sqrt{L}}{T} \\ &= L^{\frac{1}{2}} T^{-1} \quad [i \text{ is dimensionless}] \end{aligned}$$

Hence the value of C depends upon the system of units.

The following are the empirical formulae, after the name of their inventors, used to determine the value of C

1. Bazin formula:

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}}$$

S.No.	Surface of Channel	Bazin's Constant (K)
1.	Smooth Cement plaster or planed wood	0.11
2.	Concrete, brick or unplanned wood	0.21
3.	Smooth rubble masonry or poor brickwork	0.83
4.	Earth channels in very good condition	1.54
5.	Earth channels in rough condition	3.17
6.	rough Dredged earth channels average condition	2.16

Where K = Bazin's Constant and depends upon the roughness of the surface of channel
 m = Hydraulic mean depth or hydraulic radius.

2. Ganguillet - Kutter formula: - The value of C is given by in MKS unit as

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i} \right) \frac{N}{\sqrt{m}}}$$

where, N = Roughness Co-efficient which is known as Kutter's constant. whose value depends upon the type of the channel surface.

i = slope of the bed

m = Hydraulic mean depth.

Some of the typical values of N are given below:

No.	Surface of channel	N (Kutter's/Manning's constant)
1.	Smooth Cement plaster or, planed wood	0.010
2.	Very smooth concrete and planed timber	0.011
3.	Smooth concrete	0.012
4.	Glazed brickwork	0.013
5.	Vitrified clay	0.014
6.	Brick surface lined with cement mortar	0.015
7.	Earth channels in best condition.	0.017
8.	Straight unlined earth channels in good condition	0.020
9.	Rivers and earth channel in fair condition.	0.025
10.	Canal and river of rough surface with weeds.	0.030

3. Manning's formula:

$$C = \frac{1}{N} m^{\frac{1}{6}}$$

where

m = Hydraulic mean depth

N = Manning's constant which is having same value as Kutter's constant for the normal range of slope and hydraulic mean depth.

Problems

1. Find the discharge through a rectangular channel 3.5m wide having depth of water 1.5m. and bed slope as 1 in 2000. Take the value of $k = 2.36$ in Bazin's formula.

Given Data:

Width of channel, $b = 3.5\text{m}$.

Depth of flow, $d = 1.5\text{m}$

Bed slope, $i = \frac{1}{2000}$

Bazin's Constant, $k = 2.36$

Using Bazin's formula given by equation, as discharge through a rectangular channel.

To find:

Discharge through a rectangular channel

Formula Used:

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}}$$

$$Q = AC\sqrt{mi}$$

Solution:

$$\text{Area, } A = b \times d = 3.5 \times 1.5 = 5.25\text{m}^2$$

$$\text{Wetted perimeter, } P = b + 2d = 3.5 + 2(1.5) = 6.5\text{m}$$

$$\text{Hydraulic mean depth, } m = \frac{A}{P} = \frac{5.25}{6.5} = 0.8$$

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}} = \frac{157.6}{1.81 + \frac{2.36}{\sqrt{0.8}}} = 35.42$$

Discharge, Q is given by equation as

$$Q = AC \sqrt{mi}$$

$$= 5.25 \times 35.42 \sqrt{0.8 \times \frac{1}{2000}}$$

$$= 3.71 \text{ m}^3/\text{s}$$

Result:

Discharge through a rectangular channel, $Q = 3.71 \text{ m}^3/\text{s}$

2. Find the discharge through a rectangular channel 3m wide having depth of water 2m and bed slope $1 \text{ in } 600$. Take the value of $N = 0.03$ in the Kutter's formula.

Given data:

width of the channel, $b = 3\text{m}$

Depth of flow, $d = 2\text{m}$

Bed slope, $i = \frac{1}{600} = 0.001667$

Kutter's formula, $N = 0.03$.

To find:

Discharge through a rectangular channel.

Formula used:

$$C = 23 + \frac{0.00155}{i} + \frac{1}{N}$$

$$1 + \left(23 + \frac{0.00155}{i} \right) \frac{N}{\sqrt{m}}$$

Solution:

$$\text{Area of flow, } A = b \times d = 3 \times 2 = 6 \text{ m}^2$$

$$\text{Wetted perimeter, } P = b + 2d = 3 + 2(2) = 7 \text{ m}$$

$$\text{Hydraulic mean depth, } m = \frac{A}{P} = \frac{6}{7} = 0.8571 \text{ m}$$

Using Ganguillet-Kutter formula

$$\begin{aligned} C &= \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left(23 + \frac{0.00155}{i}\right) \frac{N}{\sqrt{m}}} \\ &= \frac{23 + \frac{0.00155}{0.000625} + \frac{1}{0.03}}{1 + \left(23 + \frac{0.00155}{0.000625}\right) \frac{0.03}{\sqrt{0.8571}}} \\ &= \underline{33.69} \approx 32.21 \end{aligned}$$

Discharge Q given by equation as

$$\begin{aligned} Q &= A C \sqrt{mi} \\ &= 6 \times 33.69 \sqrt{0.8571 \times 0.000625} \\ &= 4.6785 \text{ m}^3/\text{s} \approx 4.47 \text{ m}^3/\text{s} \end{aligned}$$

Result:-

Discharge through a rectangular

channel, $Q = 4.6785 \text{ m}^3/\text{s}$.

3. Find the discharge through a rectangular channel of width 2m having a bed slope of 4 in 8000. The depth of flow is 1.5m and take the value of N in Manning's formula as $N = 0.012$.

Given data:

Width of the channel, $b = 2\text{m}$

Depth of the flow, $d = 1.5\text{m}$

bed slope, $i = \frac{4}{8000} = 0.0005$

Manning's constant $N = 0.012$

Formula used: Manning's formula, $C = \frac{1}{N} \cdot m^{1/6}$

Discharge, $Q = A \times C \sqrt{mi}$

Solution: Area, $A = b \times d = 2 \times 1.5 = 3\text{ m}^2$

wetted perimeter, $P = b + 2d = 2 + 2(1.5) = 5\text{m}$

Hydraulic mean depth, $m = \frac{A}{P} = \frac{3}{5} = 0.6$

$$\therefore C = \frac{1}{0.012} (0.6)^{1/6} = 76.53$$

$$\begin{aligned} Q &= A \times C \sqrt{mi} \\ &= 0.6 \times 76.53 \sqrt{0.6 \times 0.0005} \\ &= 3.97 \text{ m}^3/\text{sec} \end{aligned}$$

Result: Discharge through rectangular channel,
 $Q = 3.97 \text{ m}^3/\text{sec}.$

Most economical cross-section for open channels

- A section of a channel is said to be most economical when the cost of construction of the channel is minimum.
- But the construction of a channel depends on excavation and the lining.
- To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum.
- This condition is utilized for determining the dimensions of economical sections of different forms of channels, i.e., best dimensions of a channel for a given area.
- The conditions to be most economical for the following shapes of the channels will be considered.

1. Rectangular channel
2. Trapezoidal channel
3. Circular channel

1. Most economical rectangular channel :-

For the most economical section is that the condition for an given area, and the perimeter' should be minimum.

Consider a rectangular channel as shown in the figure.

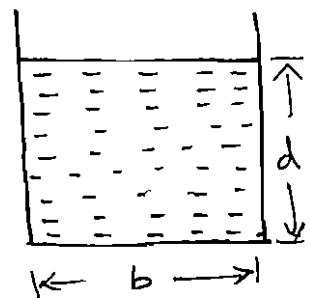
Let

b = Width of channel

d = Depth of flow

Area of flow, $A = b \times d \rightarrow \textcircled{1}$

wetted perimeter, $P = b + 2d \rightarrow \textcircled{2}$



From equation ①, $b = \frac{A}{d}$.

Substituting the value of b in Eq ②

$$\therefore P = \left(\frac{A}{d}\right) + 2d \longrightarrow \textcircled{3}$$

For the most economical section P should be minimum for a given area,

$$\text{or } \frac{dP}{d(d)} = 0$$

Differentiating the equation ③ w.r.t. 'd' and equating the same to zero, we get

$$\frac{d}{d(d)} \left[\frac{A}{d} + 2d \right] = 0 \quad \textcircled{\text{or}} \quad -\frac{A}{d^2} + 2 = 0 \quad \textcircled{\text{or}} \quad A = 2d^2$$

But

$$A = b \times d$$

$$\therefore b \times d = 2d^2 \quad \underline{\text{or}} \quad b = 2d$$

$$\left(\because \begin{array}{l} A = bd \\ P = b + 2d \end{array} \right)$$

$$\text{Now hydraulic mean depth, } m = \frac{A}{P} = \frac{b \times d}{b + 2d}$$

$$m = \frac{2d \times d}{2d + 2d} = \frac{2d^2}{2 \times d} = \frac{d}{2}$$

From the above equations shows that rectangular channel will be the most economical when

i, Either $b = 2d$ means width is two times depth of flow.

ii, or $m = \frac{d}{2}$ means hydraulic mean depth is half the depth of flow.

Problem

A rectangular channel 6m wide has depth of water 2.5m. The slope of the bed of the channel is 1 in 1000 and value of Chezy's constant $C = 55$. It is desired to increase the discharge to a maximum by changing the dimensions of the section for constant area of cross-section, slope of the bed and roughness of the channel. Find the new dimensions of the channel and increase in discharge.

Given data:

Width of channel, $b = 6\text{m}$

Depth of flow, $d = 2.5\text{m}$

Area of flow, $A = b \times d = 6 \times 2.5 = 15\text{m}^2$

Slope of bed, $i = \frac{1}{1000}$

Chezy's constant, $C = 55$

Wetted Perimeter, $P = b + 2d = 6 + 2(2.5) = 11\text{m}$

Hydraulic mean depth, $m = \frac{A}{P} = \frac{15}{11} = 0.545 \text{ (circled)} = 1.36\text{m}$

To find:

1. Maximum discharge, $Q_1 = 30.525\text{m}^3/\text{s}$

2. Increase in discharge

Formula used:

$$Q = A C \sqrt{m i}$$

$$Q_1 = A C \sqrt{m_1 i}$$

→ The discharge, Q is given by

$$Q = A C \sqrt{m i} \rightarrow \textcircled{1}$$

$$= 15 \times 55 \sqrt{\frac{0.545}{1.36} \times \frac{1}{1000}} = 19.259\text{m}^3/\text{s}$$

For maximum discharge for a given area, slope of bed and roughness we process as:

Let b_1 = new width of channel
 d_1 = new depth of flow.

Then, Area $A = b_1 \times d_1$, where $A = \text{Constant} = 15 \text{ m}^2$

$$\therefore b_1 d_1 = 15 \rightarrow (2)$$

Also for maximum discharge, $b_1 = 2d_1 \rightarrow (3)$

Substituting the value of b_1 in equation (2), we have

$$2d_1 \times d_1 = 15 \quad \text{or} \quad d_1^2 = \frac{15}{2} = 7.5$$

$$\therefore d_1 = \sqrt{7.5} = 2.738.$$

Substituting the value of d_1 in equation (3), we get

$$b_1 = 2d_1 = 2 \times 2.738 = 5.477 \text{ m}$$

New dimensions of the channel are

$$\text{Width, } b_1 = 5.477 \text{ m.}$$

$$\text{Depth, } d_1 = 2.738 \text{ m}$$

$$\text{wetted perimeter, } P_1 = b_1 + 2d_1 = 5.477 + 2(2.738)$$
$$P_1 = 10.953 \text{ m.}$$

$$\therefore \text{Hydraulic mean depth, } m_1 = \frac{A}{P_1} = \frac{15}{10.953} = 1.369 \text{ m}$$

New hydraulic mean depth, m_1 , corresponds to the condition of maximum discharge. And hence also equal to

$$\frac{d_1}{2} = \frac{2.738}{2} = 1.369 \text{ m.}$$

Maximum discharge Q_1 is given by $Q_1 = A C \sqrt{m_1 i}$

$$\therefore Q_1 = 15 \times 55 \sqrt{1.369 \times \frac{1}{1000}} = 30.525 \text{ m}^3/\text{s}$$

$$\therefore \text{Increase in discharge, } = Q_1 - Q_2 = 30.525 - 19.259$$

$$= 11.266 \text{ m}^3/\text{s}$$

Result:

Maximum discharge, $Q_1 = 30.525 \text{ m}^3/\text{s}$

Increase in discharge, $Q_1 - Q_2 = 11.266 \text{ m}^3/\text{s}$

2. Most economical Trapezoidal channel :

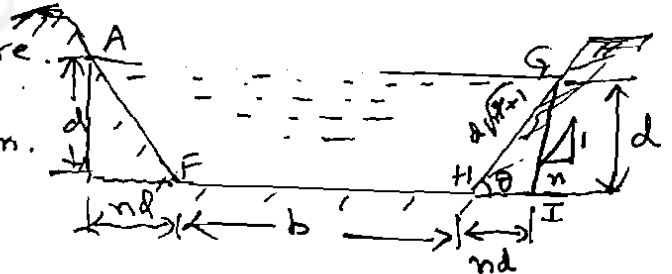
This section of a channel will be more economical, when its wetted perimeter is minimum.

Lets consider a trapezoidal section of a channel as shown in figure.

b = width of channel at bottom.

d = depth of flow

θ = Angle made by the sides with horizontal.



$\therefore$ The side slope is given as 1 vertical to n horizontal.

$$\text{Area of flow, } A = \frac{(FH + AG)}{2} \times d = \frac{b + (b + 2nd)}{2} \times d$$

$$= \frac{2b + 2nd}{2} \times d = (b + nd) d \rightarrow (1)$$

$$\frac{A}{d} = b + nd$$

$$\therefore b = \frac{A}{d} - nd$$

Now, wetted perimeter, $P = AF + FH + HG = FH + 2HG \rightarrow$

$$= b + 2\sqrt{HI^2 + GI^2}$$

$$= b + 2\sqrt{n^2d^2 + d^2}$$

$$= b + 2d\sqrt{n^2 + 1} \rightarrow (2)$$

Substituting the value of b from eq (2), we get

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \rightarrow (3)$$

For most economical section P , should be minimum

$$\frac{dP}{d(d)} = 0$$

$\therefore$ Differentiating equation (3) w.r.t. ' d ' and equating it equal to zero, we get

$$\frac{d}{d(d)} \left[\frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \right] = 0$$

$$\text{or } \frac{A}{d^2} + n - 2\sqrt{n^2 + 1} = 0 \quad [\because n \text{ is constant}]$$

$$\text{or } \frac{A}{d^2} + n = 2\sqrt{n^2 + 1}$$

Substituting the value of A from eq (1) in the above equation.

$$\frac{(b + nd)d}{d^2} + n = 2\sqrt{n^2 + 1}$$

$$\frac{b + nd}{d} + \frac{n}{1} = 2\sqrt{n^2 + 1}$$

$$\text{or } \frac{b + nd + nd}{d} = \frac{b + 2nd}{d} = 2\sqrt{n^2 + 1}$$

$$3) \quad \frac{b+2nd}{2} = d \sqrt{n^2+1} \rightarrow (4)$$

But from the given figure $\frac{b+2nd}{2} = \text{Half of top width.}$

$d \sqrt{n^2+1} = H.G. = \text{one of the sloping side}$

Equation (4) is the required conditions for a trapezoidal section to be most economical, which can be expressed as half of the top width must be equal to one of the sloping sides of the channel.

iii, Hydraulic mean depth.

Hydraulic mean depth, $m = \frac{A}{P}$

Value of A from (1), $A = (b+nd) d$

Value of P from (2), $P = b + 2d \sqrt{n^2+1}$

$$= b + 2 \times \frac{(b+2nd)}{2} \quad (\because \text{from eq. (4)})$$

$$= 2b + 2nd$$

$$= 2(b+nd)$$

$$\therefore \text{Hydraulic mean depth, } m = \frac{A}{P} = \frac{(b+nd) d}{(b+nd) 2}$$

$$m = \frac{d}{2}$$

Hence for a trapezoidal section to be most economical hydraulic mean depth must be equal to half the depth of flow.

For the trapezoidal section of most economical section are tangential to the semi circle describing the water line. This is proved as:

The given figure shows the trapezoidal channel of most economical section.

Let

θ = Angle made by the sloping side with horizontal, and

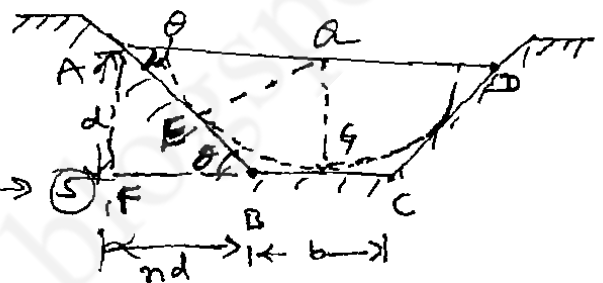
Q = The centre of the top width, AD

Draw QE perpendicular to the sloping side AB

$\triangle AEF$ is a right-angled triangle and $\angle AEF = \theta$

$$\therefore \sin \theta = \frac{QE}{QA}$$

$$\therefore QE = AQ \sin \theta \rightarrow (5)$$



$$\text{In } \triangle AFB, \sin \theta = \frac{AF}{AB} = \frac{d}{d\sqrt{n^2+1}} = \frac{1}{\sqrt{n^2+1}}$$

Substituting $\sin \theta = \frac{1}{\sqrt{1+n^2}}$ in eq. (5), we get

$$\therefore QE = AQ \times \frac{1}{\sqrt{1+n^2}} \rightarrow (6)$$

But AQ = Half of top width

$$= \frac{b+2nd}{2} = d\sqrt{n^2+1} \rightarrow (4)$$

Substituting the value of AQ in eq. (6)

$$\therefore QE = \frac{d\sqrt{n^2+1}}{\sqrt{n^2+1}} = d \text{ (depth of flow)}$$

Most economical circular channel :

→ In the circular channel, the area of flow cannot be maintained constant.

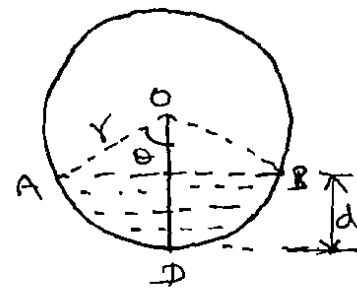
→ With the change of depth of the flow in a circular channel of any radius, the wetted area and wetted perimeter have changes.

→ For the most economical section, two separate conditions are obtained. They are

1. Condition for maximum velocity, and
2. Condition for maximum discharge.

1. Condition for maximum velocity for circular section.

The given figure shows a circular channel through which water is flowing.



Let d = depth of water

θ = Angle subtended at the Centre by water surface.

r = radius of channel and

s = slope of the bed

The velocity of flow according to chezy's formula

is given as $V = C \sqrt{mi} = C \sqrt{\frac{A}{P} i}$ ($\because m = \frac{A}{P}$)

The maximum flow will be obtained through the circular channel, when the hydraulic mean depth 'm' or $\frac{A}{P}$ is maximum for a given C and i.

Hence for maximum value of $\frac{A}{P}$ we have condition

$$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0 \rightarrow (1)$$

where A and P both are functions of θ ,

The value of wetted ~~perimeter~~ ^{Area A}, is given by equation as

$$A = r^2 \left(\theta - \frac{\sin 2\theta}{2} \right) \rightarrow (2)$$

The value of wetted Perimeter, P is given by equation as

$$P = 2r\theta \rightarrow (3)$$

Differentiating equation on (1) w.r. to θ , we have

$$\frac{P \frac{dA}{d\theta} - A \frac{dP}{d\theta}}{P^2} = 0$$

or $P \cdot \frac{dA}{d\theta} - A \cdot \frac{dP}{d\theta} = 0 \rightarrow (4)$

From eq. (2), $\frac{dA}{d\theta} = r^2 \left(1 - \frac{\cos 2\theta}{2} \times 2 \right) = r^2 (1 - \cos 2\theta)$

From eq. (3), $\frac{dP}{d\theta} = 2r$

Substituting the values of $\frac{dA}{d\theta}$ and $\frac{dy}{d\theta}$ in eq (4)

$$2r\theta \left[r^2 (1 - \cos 2\theta) \right] - r^2 \left(\theta - \frac{\sin 2\theta}{2} \right) (2r) = 0$$

$$\text{or } 2r^3 \theta (1 - \cos 2\theta) - 2r^3 \left(\theta - \frac{\sin 2\theta}{2} \right) = 0$$

$$\theta (1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2} \right) = 0 \quad (\text{cancelling } 2r^3)$$

$$\theta - \theta \cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

$$\text{or } \theta \cos 2\theta = \frac{\sin 2\theta}{2} \quad \text{or } \frac{\sin 2\theta}{\cos 2\theta} = 2\theta$$

$$\therefore \tan 2\theta = 2\theta$$

The solution of this equation by trial, it gives

$$2\theta = 257^\circ 30'$$

$$\theta = 128^\circ 45'$$

or The depth of flow for maximum velocity from figure

$$d = OD - OC = r - r \cos \theta$$

$$= r [1 - \cos \theta] = r [1 - \cos 128^\circ 45']$$

$$= r [1 - \cos (180^\circ - 51^\circ 15')]$$

$$= r [1 - (-\cos 51^\circ 15')]$$

$$= r [1 + \cos 51^\circ 15']$$

$$= r [1 + 0.62] = 1.62 r = 1.62 \frac{D}{2}$$

$$= \underline{\underline{0.81D}}$$

Where D = diameter of the circular channel

Hence the depth of water in the circular channel should be equal to 0.81 times the diameter of the channel.

Hydraulic mean depth for maximum velocity is

$$m = \frac{A}{P} = \frac{r^2 \left(\theta - \frac{\sin 2\theta}{2} \right)}{2r\theta} = \frac{r}{2\theta} \left[\theta - \frac{\sin 2\theta}{2} \right]$$

Where $\theta = 128^\circ 45' = 128.75^\circ$

$$= 128.75 \times \frac{\pi}{180} = 2.247 \text{ radians.}$$

$$\begin{aligned} m &= \frac{r}{2 \times 2.247} \left[2.247 - \frac{\sin 257.30^\circ}{2} \right] \\ &= \frac{r}{4.494} \left[2.247 - \frac{\sin (180 + 77.30^\circ)}{2} \right] \\ &= \frac{r}{4.494} \left[2.247 + \frac{\sin 77.30^\circ}{2} \right] = 0.611 r \\ &= 0.611 \times \frac{D}{2} = 0.33055 D = 0.33 D \end{aligned}$$

Thus for maximum velocity, the hydraulic mean depth is equal to 0.33 times the diameter of circular channel.

Condition for maximum discharge for circular section.

The discharge through a channel is given by

$$Q = AC \sqrt{m_i} = AC \sqrt{\frac{A}{P} i} = C \sqrt{\frac{A^3}{P} i}$$

The discharge will be maximum for constant values of C and i , when $\frac{A^3}{P}$ is maximum. $\frac{A^3}{P}$ will be maximum

when $\frac{d}{d\theta} \left(\frac{A^3}{P} \right) = 0$.

Differentiating this equation with respect to θ and equating the same to zero, we get

$$\frac{P \cdot 3A^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta}}{P^2} = 0$$

$$3PA^2 \frac{dA}{d\theta} - A^3 \frac{dP}{d\theta} = 0$$

Dividing by A^2 , $3P \frac{dA}{d\theta} - A \frac{dP}{d\theta} = 0 \rightarrow \textcircled{1}$

But from equation $P = 2r\theta$

$$\frac{dP}{d\theta} = 2r$$

From equation, $A = r^2 \left(\theta - \frac{\sin 2\theta}{2} \right)$

$$\frac{dA}{d\theta} = r^2 (1 - \cos 2\theta)$$

Substituting the values of P , A , $\frac{dP}{d\theta}$ and $\frac{dA}{d\theta}$

in eq. (1).

$$3 \times 2r\theta \times r^2 (1 - \cos 2\theta) - r^2 \left(\theta - \frac{\sin 2\theta}{2} \right) \times 2r = 0$$

$$6r^3 \theta (1 - \cos 2\theta) - 2r^3 \left(\theta - \frac{\sin 2\theta}{2} \right) = 0$$

Dividing by $2r^3$, we get

$$3\theta (1 - \cos 2\theta) - \left(0 - \frac{\sin 2\theta}{2}\right) = 0$$

$$3\theta - 3\theta \cos 2\theta - 0 + \frac{\sin 2\theta}{2} = 0$$

The solution of this equation by trial, it gives

$$2\theta = 308^\circ$$

$$\theta = \frac{308}{2} = 154^\circ$$

Depth of flow for maximum discharge

$$d = OD - OC = r - r \cos \theta$$

$$= r [1 - \cos \theta] = r [1 - \cos 154^\circ]$$

$$= r [1 - \cos (180 - 26^\circ)]$$

$$= r [1 + \cos 26^\circ]$$

$$= 1.898$$

$$= 1.898 \times \frac{D}{2} = 0.948D = 0.95D$$

Where, D = Diameter of Circular channel

Thus for maximum discharge through a circular channel

the depth of flow is equal to 0.95 times its diameter.

Specific energy

* The energy of a flowing fluid per unit wt is given by

$$\text{total energy} = z + h + \frac{V^2}{2g}$$

where

z = height of the bottom of channel above datum

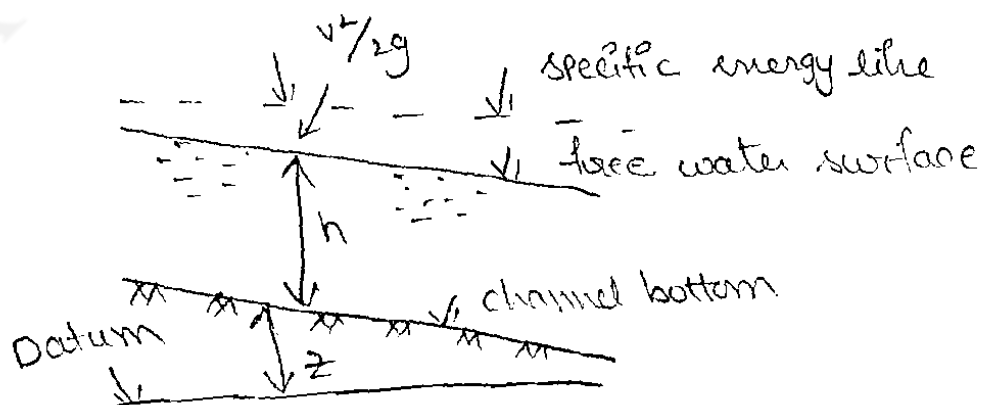
h = depth of fluid

V = mean velocity of flow

* If the channel bottom is taken as the datum as shown in fig, then the total energy per unit weight of fluid will be

$$E = h + \frac{V^2}{2g}$$

* Derived equation shows the specific energy, hence specific energy of a flowing fluid is defined as energy per unit weight of the fluid with respect to the bottom of the channel



Specific energy curve 6-

* The specific energy (E) of a fluid is defined as the energy per unit wt of water measured with respect to the bottom of the channel. For channel with small slope, specific energy can be expressed as

$$E = y + \frac{V^2}{2g}$$

where

E = specific energy (m)

y = depth of water (m)

$\frac{V^2}{2g}$ = velocity head (m)

The velocity head $\frac{V^2}{2g}$ can be expressed as $\frac{Q^2}{2gA^2}$

Q = flow rate (m^3/sec) and

A = cross sectional area of flow (m^2)

$$\therefore E = \frac{Q^2}{2gA^2}$$

* The specific energy can also be expressed in terms of unit width of channel by using $q = Q/b$

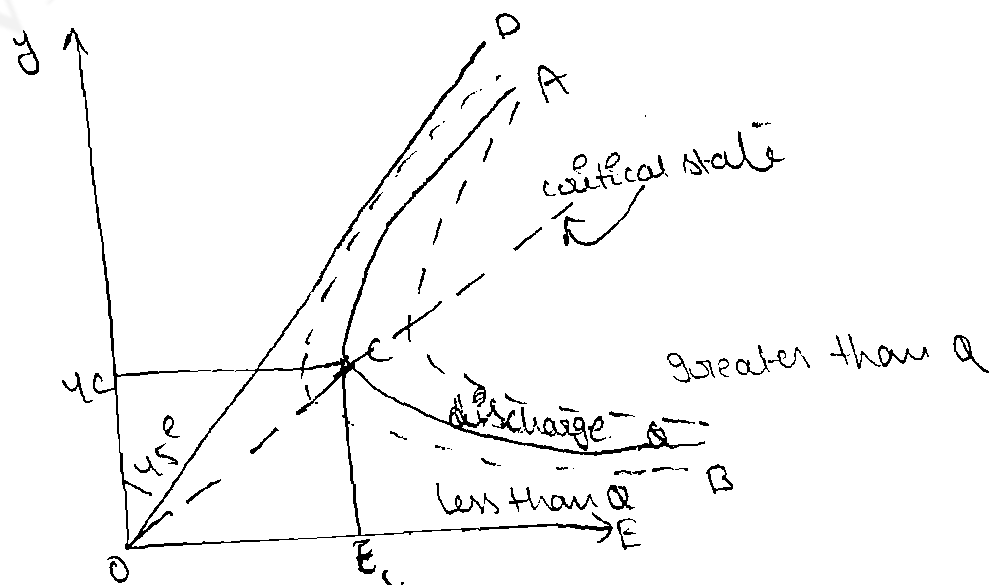
q = the flow rate per unit width of the channel

b = width of the channel (m)

* Equation (3) is the particular case of the specific energy for a rectangular channel in terms of flow rate per unit width

$$E = y + \frac{v^2}{2gy^2}$$

- * The given figure represents the following characteristics
- The specific energy curve is asymptotic to the line $y = E$ which is a 45° line (OD)
- The velocity head is equal to the horizontal distance between the 45° line and the specific energy curve
- The specific energy diagram shows that the specific energy is minimum at critical depth at a point C (at critical depth)
- It is possible to have two depths of water for a particular specific energy (different from the minimum specific energy); a subcritical depth and a supercritical depth
- For a given channel, different flow rates will give a family of specific energy curves
- The line that joins the minimum specific energy points of each curve defines the critical state



Critical Depth (h_c)

Critical depth is defined as that depth of flow of water at which the specific energy is minimum. This is denoted by ' h_c '

$$E = y + \frac{v^2}{2gy^2} \text{ from the eq (1)}$$

$$(or) \frac{dE}{dh} = 0$$

$$\left[\because \frac{v^2}{2g} \text{ is constant} \right]$$

$$(or) \frac{d}{dh} \left[y + \frac{v^2}{2gy^2} \right] = 0 \quad (or) \quad 1 + \frac{v^2}{2g} \left[\frac{-2}{y^3} \right] = 0$$

$$(or) \quad 1 - \frac{v^2}{gy^3} = 0 \quad (or) \quad 1 = \frac{v^2}{gy^3} \quad (or) \quad y^3 = \frac{v^2}{g}$$

$$h = \left[\frac{v^2}{g} \right]^{1/3}$$

while when the specific is minimum depth is critical and it is denoted by h_c hence critical depth is

$$h_c = h = \left[\frac{v^2}{g} \right]^{1/3}$$

types of critical flow

1. critical flow :-

when the specific energy is under minimum level (or) otherwise the flow corresponding to the critical depth is defined as critical flow

$$V_c = \sqrt{g \times h_c} \quad \text{or} \quad \frac{V_c}{\sqrt{g h_c}} = 1$$

where $\frac{V_c}{\sqrt{g h_c}} = \text{Froude number}$

$F_e = 1.0$ for critical flow

2. Sub-critical flow :-
~~~~~

When the depth of flow in a channel is greater than the critical depth ( $h_c$ ) then the flow is said to be subcritical flow. For this type of flow the Froude number is less than one

$$F_e < 1.0$$

3. Super-critical flow :-  
~~~~~

When the depth of flow in a channel is less than the critical depth (h_c) the flow is said to be super-critical flow. For this type of flow the Froude number is greater than one

$$F_e > 1.0$$

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